ON THE VARIATION IN THE EXPERIMENTALLY DETERMINED VALUES OF THE MESON MASS

By D. M. BOSE *

AND

Miss BIBHA CHOUDHURI

(*Received for Publication, September 9, 1944*)

ABSTRACT. In this paper all reliable data on meson mass determinations have been collected together. The assumptions underlying the different experimental methods as well as their reliability have been discussed. It is found that large variations in the meson mass values occur, both when different experimental methods are used as well as when the same method is used by different observers. It is made probable that apart from the large errors of measurements associated with the present methods of meson mass determinations, the variation in mass values depend also in some way not representable by the relativity formula on the velocity of the meson particle. Possible causes of such variations are discussed.

1. INTRODUCTION

In a previous paper by one of us, (Choudhuri 1944), (referred to as Paper I), a method is described of measuring the mass of cosmic ray particles which produce single ionisation tracks on photographic plates exposed under air to cosmic rays at Sandakphu (elevation 12,000 ft.). The results of measurements made on two plates exposed at different times to cosmic radiation is given in Table I.

Plates I and II were Ilford new halftone plates, taken from two different batches ordered from England at intervals of one year, and sent to Sandakphu on two different occasions between which one year intervened. Plate I recorded a larger number of ionization tracks than Plate II; in the latter very few tracks were found in which the mean grain spacing between silver grains deposited along them lay between 5 and 6μ. In spite of the differences in the sensitiveness of the two plates, the average mass of the penetrating particles had very similar values and varied in a similar way with the energy. Such agreement between the mass values as function of the mean energy of the particles, appeared to us to preclude the possibility of the mass variation being due to statistical errors. Starting from low energy particles, it will be noticed that the average mass diminishes with increasing energy of the particles to a minimum, and then again it increases.

It is well-known that wide variations in the experimentally determined meson mass values have been obtained by competent observers, using different experimental methods. Even the same experimental method used by different investigators have led on different occasions to widely differing mass values. Such variations, in the results obtained by one and the same observer may be due partly to experimental difficulties, and in the results of different investigators using the same technique, to the different methods of interpreting their experimental observations. One good example of such differences in interpretation is the different empirical methods used by Williams and Wilson (1939), and by Corson and Brode (1938) of deducing the velocity βc of the ionising particles as function of the relative ionization density D along their tracks (see section 3 A).

* Fellow of the Indian Physical Society.

But even making allowances for the occurence of large experimental errors, the mass values obtained are so widely different that it has led competent investigators like Anderson and Neddermeyer (1939) to the statement "it has become increasingly likely that a complete interpretation of the experimental data is not found in the single assumption of an unstable particle with a single charge and a unique mass of the order of 200 electron mass".

Another characteristic constant of the meson, *viz.*, its proper life time τ_0. also shows large variations in experimentally determined values. Weisz (1941) has recalculated the rest life time of cosmic ray mesons obtained by different observers, on the basis of the same rest mass. The values of the rest time thus recalculated still disagree amongst themselves and they are shown by Weisz to be function of the mean path-length of the decaying meson employed in the different experiments. Weisz says " when instead of the existence of only one possible value of the rest mass, for which we have no experimental evidence, a distribution of rest mass is assumed, the described phenomena can be accounted for, regardless of what form the mass distribution may have". The assumption made by Weisz is 'that at each energy there is a distribution of rest mass amongst the mesons". Bernadini (*et al* 1941) conclude their recent short communication "Differential measurement of meson life time at different elevations" with the remark " we wish to call attention to our results that $(\mu c^2)/\tau$ increases with increased altitude and *average meson energy*. " Since $(\mu c^2)/\tau = (\mu c^2)/\tau_0 \sqrt{1-\beta^2}$ this quantity should diminish with increasing β, *i.e.*, if the interpretation of their experimental results by these investigators is valid, then it would imply that the rest mass of the meson increases with its energy in a manner not accountable by the theory of relativity.

With such variety of results and of their interpretations, on the existence and possible cause of the variation in the rest mass of the meson, it seemed worthwhile to examine the existing experimental data on meson mass measurement and to find out whether any correlation can be found between the kinetic energies of the meson particles and their rest mass values.

The experimental data discussed in this paper are taken from

(*i*) a collection of results of twenty-four mass determinations of Wheeler and Ladenburg (1941).

(*ii*) additional four mass determinations recently published by Nielson and Powell (1943).

(*iii*) five mass determinations contained in Paper I.

2. THEORY

The methods which have so far been employed for determining the mass of the penetrating cosmic ray particles are based upon observations on the tracks of such particles in Wilson cloud chamber. Only in the method developed by us have observations been made on tracks of such particles in photographic emulsion. The principal characteristics of such ionising particles are its charge Ze, mass μ and velocity v. Under certain special circumstances, *e.g.*, during close collisions of such high energy particles with atomic nuclei, the other characteristic constants of the particles, *viz.*, its spin and magnetic moment become effective. Further during close collisions interaction of non-electrical nature can occur. For our present purpose the knowledge of the first three constants are sufficient. It is an observed fact that, barring some results of Schopper (1939) on tracks of cosmic ray particles in photographic emulsions, all the cosmic ray particle tracks with β near unity, have approximately the same ionization density, from which the conclusion is drawn that these cosmic ray particles carry the same charge e. Using this assumption it is necessary to make two independent observations for the determination of the mass of the penetrating particles. For all Wilson chamber observations, one such determination is the curvature of the tracks of the particles in magnetic fields : the latter is given by the equation

$$pc = He\rho, \text{ where } p = \frac{\mu\beta c}{\sqrt{1-\beta^2}} \quad \ldots \quad (1)$$

The other measurement is usually based upon the loss of energy suffered by the penetrating particles in the media traversed by them. The specific charge on these particles is such, that radiation loss of energy is negligible.

The ionization of such singly charged particles is given by the equation

$$\frac{dE}{dx} = \frac{2\pi NZe^4}{mc^2\beta^2}\left[\log\frac{mc^2\beta^2 W_m}{(1-\beta^2)I^2Z^2} + (1-\beta^2)\right] \quad \ldots \quad (2)$$

Where W_m is the maximum energy transferred by collision from the moving particle of mass μ and velocity βc to an electron, and I is the average excitation potential of the atoms traversed by the particle. The calculations of the energy loss is rather complicated when we come to the relativistic region. Ladenburg and Wheeler (1941) have shown that the expression for the energy loss takes a simple form

$$\frac{dE}{dx} = \frac{A}{\beta^2}\left[ln\ K + ln\frac{\beta^2}{1-\beta^2} + (1-\beta^2)\right] \quad \ldots \quad (3)$$

when the following approximations are made. (*a*) The mass μ of the primary particle is large compared to the electron mass, (*b*) its energy is small compared to

$$\left(\frac{\mu}{m}\right)\mu c^2,$$

(c) it is moving faster than the bound electrons in the stopping atoms, and (d) its capture and loss of electron can be neglected and its nuclear charge is not very large. It will be seen that the mass and the energy of the penetrating particles used for mass determinations lie in such regions that these criteria are generally satisfied ; only in the case of the passage of heavy particles in photographic emulsions, it was shown by one of us (Choudhuri, 1942) that the conditions (*c*) & (*d*) are not satisfied,

3. METHODS

We shall now discuss the different experimental methods.

Method A. Ionization loss and curvature—The largest number of mass determinations has been made from measurements of the curvature of penetrating particle tracks in a magnetic field, together with a count of the average number of ions per cm length of the track. The function on the right hand side of (3) varies as $1/\beta^2$ for $\beta<0.9$ and has a minimum value for $\beta=0.97$ and then increases with increasing β. The ionization corresponding to this minimum value is denoted by I_0. For such measurements only particles are selected for which $\beta<0.97$. We define a quantity $D=I/I_0$ as the relative specific ionization, and the experimental technique consists in deducing the value of β from measurements of D.

For this purpose two methods have been used

(*a*) Wilson and Williams (1939) have counted the number of distinct groups of droplets per cm length in sharply defined Wilson chamber tracks of low energy electrons. Each group correspond to one primary ion pair. They represent the number of ions produced by the empirical formula $I=I_0\beta^{-1\cdot4}$. According to Corson and Brode (1938) the theoretical expression (3) can well be represented in the interval $0.2<\beta<0.9$ by a similar expression $I=I_0\beta^{-1\cdot8}$. The mass values determined by the empirical curve are larger than those obtained from the theoretical curve.

(*b*) In the method used by Corson and Brode (1938) the expansion of the chamber is delayed, to enable the oppositely charged ions produced along the track of the ionizing particle to separate under the influence of an applied electric field. The measurements then made do not give the primary ionization, but the probable ionization, *i.e.*, the primary ionization plus the ionization produded by secondaries of energy lower than a certain critical energy. The minimum probable ionization was found to be 50 ions per cm track in air, O_2 and N_2 at N.T.P. Corson and Brode have constructed a nomograph, from which knowing the value of $H\rho$ and D the value of μ can be obtained.

In Table II A are collected ail the data on the meson mass determinations using the above method. In view of the large discrepancy in the values obtained by different observers, and the different methods used by them to correlate D with β we have grouped separately the results of the measurements of (*i*) William and Pickup, (*ii*) Nielson and Powell, and (*iii*) also a number of isolated measurements by other investigators. In each group the data are arranged in order of increasing kinetic energies of the particles. The kinetic energies of these particles given in column I are calculated from their momenta, on the assumption that the mesotron mass is 200 m_0.

Method B. Curvature and Range—This is a convenient method as it does not involve measurement of specific ionisation loss and is therefore independent of any special assumptions underlying the ionisation loss formula. Since the loss of energy per cm of track of all particles carrying the same charge, depend only on their velocities we can write the range $R=k.\ m.f(v)$; and for all particles starting with the same initial velocity $\frac{R_1}{R_2}=\frac{m_1}{m_2}$. If we take the second particle a proton, we can, by putting a reasonable value for m_1, find the value R_2 the range of a proton particle starting with the same initial velocity.

βc. The value of βc for a proton with a given range can be obtained from the empirical curves given by Livingston and Bethe (1937). The correct value $m_1 = \mu$ will be such as to satisfy the equation $\frac{\mu \beta c}{\sqrt{1-\beta^2}} = He\rho$; where $H\rho$ is obtained experimentally. Corson and Brode (*loc. cit.*) consider this to be the most accurate method of measuring mass, provided all the data are given with the same degree of precision. But as Ladenburg and Wheeler (1941) point out, the drawback of the method is that as the particle slows down it is subject to increasing amount of scattering which it is difficult to make allowances for. Thus it is found that the largest amount of discrepancy occur amongst the mass determinations by this method. The results are given in Section B of Table II. The results obtained by Maier Leibnitz and some obtained by Anderson and Neddermeyer have been omitted as being widely outside the probable range of values of the meson mass, which we take to vary between 150 m_0 – 300 m_0.

Method C. Curvature and Momentum loss—The change of momentum suffered by a fast particle after traversing a heavy metal plate of known thickness can be used for accurate measurement of mass in the non-relativistic region according to Ladenburg and Wheeler. The change of momentum suffered after traversal through a thickness Δx of the given plate is $\Delta p = \frac{He}{c} . \Delta\rho$ from which we have the relation $\frac{\Delta E}{\Delta x} = c\beta \frac{\Delta p}{\Delta x}$. This value can be introduced in equation (2) or (3) to get the value of β. The resolving power of the method is poor for $H\rho >> 4 \times 10^5$ gauss cm. The mass determinations according to this method are given in section C of Table II. Nishina originally determined the meson mass using Bloch's form of the energy loss formula and found the value of μ to be between 180-260 m_0. Later on using Bhabha's formula for relativistic loss, he found the value of μ to be (180 ± 20) m_0. The energy of his particle is $\sim 10^8 eV$ and falls within the non-relativistic range, hence there appears to be no justification for his using the relativistic energy loss formula.

Method D. Curvature and electron collision—The recoiling electron receives such a large amount of energy from the colliding primary particle, that it can produce an ionization track of at least a centimeter length. The energy or momentum of the recoiling electron can be determined either from its range or curvature in the magnetic field.

Method E. Photographic plate method—Here the mean ionisation along a long meson track and its curvature due to multiple scattering are determined. The average number of silver grains deposited per unit length of the cosmic ray particles is assumed to be proportional to the initial kinetic energy of the particle. Further it is assumed that both the meson and the proton carry one unit of charge, so that when these two particles start with the same velocity, the mean grain number deposited along their tracks will be the same. The mean kinetic energy of a number of cosmic ray particles which have the same mean grain number can be deduced from the mean curvature of their tracks in the photographic emulsion due to multiple scattering. In this method the ratio of the mass of the unknown particle to that of the proton is determined from the ratio of the kinetic energies of these two particles which have the same mean grain spacing along their respective tracks in a given photographic emulsion (Table I).

4. DISCUSSIONS.

It will be noticed that in spite of large variations in the values of the meson mass obtained, (*i*) by using different experimental methods and (*ii*) by different investigators using the same method, that certain general trend in the variation of the meson mass with the kinetic energy of the particles appear. For example it is found that for higher velocities the meson mass, as determined by the different methods, appear to increase with the particle energy, in a way not representable by the relativity formula. Further it also appears, from a consideration of the results obtained by the methods A and E, and they contain the largest number of measurements, that starting with the low velocity particles, the measured values of the meson mass diminish initially with the particle energy to a minimum value, which appear to lie in the region of 6 to $10 \times 10^6 eV$ in the case of Wilson chamber meson tracks, and of $1.0 \times 10^6 eV$ in the case of meson tracks in photographic emulsion. That this variation is dependent on the particle energy is shown by the measurements made on photographic tracks. Each mass measurement is based upon summation taken over track lengths varying from 10 to 33 in number. This is contrary to the assumption made by Weisz (1941) "that at each energy there is a distribution of rest mass." We shall next discuss the possible explanations which can be put forward to explain such observed variations in the meson mass.

(*a*) The energy loss equation is correct, but the assumptions underlying the deduction of formula (3) are not satisfied, *viz.*, (*i*) the energy of the particle is small compared to $\mu c^2 \left(\frac{\mu}{m_0} \right)$ which for mesons is $\sim 10^{10}$ eV and (*ii*) the particle velocity is large compared to those of the bound electrons of the atoms traversed by the particle. The first condition is satisfied by all the penetrating particles used in the mass determinations, and so also the second condition.

It was shown by one of us (Choudhury, 1942) that the second condition was not satisfied in the case of α-particles of energy between 5 to 10×10^6 eV. It was observed that the ratio of track lengths of α-particles of a given energy in photographic emulsion/air, came to 7×10^{-4} against a theoretically expected value of 5×10^{-4}. This was explained by us as being due to the presence of heavy atoms like Ag, Br, etc., in the photographic emulsion, in respect of some of whose inner bound electrons condition (*ii*) is not satisfied by the particles.

(*b*) The theoretical ionisation loss formula does not correctly represent experimental facts. Against this it may be pointed out that the methods B and E do not make use of the ionisation loss formula. In both use is made of the empirically determined relation between the range and energy of protons in air and in photographic emulsions.

TABLE I

Plate	m. g. s. in μ	No. of Tracks	Mean energy	Mass in units of m_0
E. I	6−5	12	2.7×10^6eV	221
...	5−4	22	1.05×10^6eV	160
...	4−3	33	0.6×10^6eV	263
II	5−4	10	1.1×10^6eV	180
...	4−3	15	0.68×10^6eV	257

TABLE II

No.	Energy	Mass	Method	Reference
A (*i*)	5.6×10^6eV	220 ± 50	Curvature and ionisation	William & Pickup, Nature 141, 684 (1938).
	5.8×10^6eV	160 ± 30	,,	ditto
	9.2×10^6	$\sim 190 \pm 60$	,,	,,
(*ii*)	5×10^6eV	225 ± 20	,,	Neilson & Powell, Phys. Rev. 63, 384 (1943)
	5.6×10^6	240 ± 15	,,	ditto
	1.15×10^7	155 ± 30	,,	ditto
	2.3×10^7	230 ± 20	,,	,,
(*iii*)	4×10^6eV	160	...	Stret & Stevenson, Phys. Rev. 53, 1003 (calculated by Corson & Brode) (1937)
	4.5×10^6eV	180 ± 25	...	Starr & Brode, Phys. Rev. 53, 3 (1938)
	1.03×10^7eV	250	...	Corson & Brode, Phy. Rev. 53, 215 (1938)
	1.6×10^7eV	200	...	Ehrenfest, Jr. C R. Paris, 206, 428 (1938).
B	$.68 \times 10^6$eV	170 ± 8	Curvature and Range	Nishina, Takeuche, Phys. Rev. 55, 585 (1939) (Considered doubtful)
	1.3×10^6eV	< 200	,,	Brode & Starr. Phy. Rev. 53, 3, (1938)
	1.3×10^6eV	~ 350	,,	Anderson & Neddermeyer, Phy. Rev. 50, 26 (1936)
	1.3×10^7eV	220 ± 35	,,	Anderson & Neddermeyer, Phys. Rev. 54, 88 (1938)
C	9×10^8eV	170 ± 20	Change in curv. due to passing through lead plate.	Wilson, Proc. Roy. Soc. 172, 521 (1939)
	1.6×10^8eV	250 ± 50		Wilson ditto
	1.8×10^8eV	$180 \sim 260$		
		180 ± 20 (recalculated)	,,	Nishina, Phys. Rev. 55, 585 (1939)
D	4.5×10^6eV	180 ± 25	Curvature and collision with electron	Hughes, Phys. Rev. 60, 414 (1941).
	4.3×10^7eV	240 ± 20		Leprince, Ringuet, Phys. Rev. 59, 460 (1941).

(*c*) The charge on the meson does not remain constant—there has been a certain amount of speculation as to the existence of neutral meson (neutretto). According to the recent theory of Hamilton, Heitler and Peng (1943), the existence of neutretto is assumed. The cross section for the transformation of a neutretto into a charged meson is taken to $\sim\epsilon^{-6}$, where ϵ is its energy. Thus at comparatively low energies of the order of 10^6-10^7 eV, there is some probability of a charged meson losing and recovering its charge by nuclear collisions, depending in some way on its velocity. If that happens the average charge on a low energy meson will be less than unity, and be some function of its velocity.

(*d*) The meson mass is not a constant, but is a function of its kinetic energy. This assumption will also support the interpretation given by Bernadini et al (1941) of their observations on the values of meson life time at different elevations.

The above discussion on the possibility of the meson mass being dependent in some unknown way on its velocity is of an exploratory nature only; its purpose is to draw attention to the desirability of undertaking further accurate investigations on mass determination of cosmic ray penetrating particles with different methods and using particles of widely varying kinetic energies. With these additional results it will be possible to find out whether such a variation of mass exists, and if so how the variation depends upon the kinetic energy of the meson particles.

BOSE RESEARCH INSTITUTE,
CALCUTTA.

REFERENCES

Anderson & Neddermeyer (1939) *Rev. Mod. Phys.*, **11**, 195.
Bernadini, G. D et al. (1941) *Phys. Rev.*, **60**, 910.
Choudhuri Bibha (1942) *Trans. Bose Inst.*, **15**, 910.
,, ,, (1944) *Ind. Jour. Phy.*, **18**, 57.
Corson & Brode (1938) *Phys. Rev.* **53**, 773.
Hamilton, Heitler, Peng (1943) *ibid.*, **64**, 78.
Ladenburg, Wheeler (1941) *ibid.*, **60**, 754.
Livingston, Bethe (1937) *Rev. Mod. Phys.*, **9**, 268.
Nielson Powell (1943) *Phys. Rev.*, **63**, 384.
Schopper and Schopper (1939) *Phys. Zeit.*, **40**, 22.
Weisz, P., (1941) *Phys. Rev.*, **59**, 845.
Wilson, Williams, E. J., (1939) *Proc. Roy. Soc. Lond. A.*, **172**, 194.